Cheng [11]. The present results are in good agreement with the values by Shah and Cheng. The fully developed Nusselt number values for the slug flow are also listed in Table 2. They differ from the values for the developed flow. Results for the Nu_{H1}, as a function of the dimensionless axial distance $[(z/D_h)/(Re\,Pr)]$, are plotted in Fig. 3 with the polygonal number n as a curve parameter. The fully developed Nusselt number values are also plotted in this figure by chain lines.

CONCLUDING REMARKS

Three-dimensional heat transfer and fluid flow characteristics in the entrance region of a polygonal duct are analyzed numerically by a coordinate transformation technique coupled with a calculation procedure for three-dimensional parabolic flows. The fully developed values of the Nusselt numbers and friction factors approach the available asymptotic results. The entry length results for the limiting case of a rectangular duct are in perfect agreement with the experimental and numerical results.

REFERENCES

1. R. K. Shah and A. L. London, *Laminar Flow Forced Convection in Ducts*. Academic Press, New York (1978).
2. S. V. Patankar and D. B. Spalding, A calculation procedure for heat, mass, and momentum transfer in three-dimensional parabolic flows, *Int. J. Heat Mass Transfer* **15**, 1787–1805 (1972).
3. C. Prakash and Ye-Di Liu, Analysis of laminar flow and heat transfer in the entrance region of an internally finned circular duct, *ASME J. Heat Transfer* **107**, 84–91 (1985).
4. K. C. Karki and S. V. Patankar, Heat transfer augmentation due to buoyancy effects in the entrance region of a shrouded fin array, *ASME J. Heat Transfer* **109**, 671–676 (1987).
5. A. Lawal and A. S. Mujumdar, Laminar flow and heat transfer in power-law fluids flowing in arbitrary cross-sectional ducts, *Numer. Heat Transfer* **8**, 217–244 (1985).
6. Y. Asako and M. Faghri, Three dimensional heat transfer and fluid flow characteristics in the entrance region of ducts of arbitrary cross-section, ASME paper 86-HT-12 (1986).
7. M. Faghri, E. M. Sparrow and A. T. Prata, Finite difference solutions of convection–diffusion problems in irregular domains using a non-orthogonal coordinate transformation, *Numer. Heat Transfer* **7**, 183–209 (1984).
8. M. Faghri and Y. Asako, Numerical determination of heat transfer and pressure drop characteristics for a converging–diverging flow channel, *ASME J. Heat Transfer* **109**, 606–612 (1987).
9. F. S. Shih, Laminar flow in axisymmetric conduits by a rational approach, *Can. J. Chem. Engng* **45**, 285–294 (1967).
10. R. K. Shah, Laminar flow friction and forced convection heat transfer in ducts of arbitrary geometry, *Int. J. Heat Mass Transfer* **18**, 849–862 (1975).
11. K. C. Cheng, Laminar flow and heat transfer characteristics in regular polygonal ducts, *Proc. Int. Heat Transfer Conf., 3rd AIChE*, New York, Vol. 1, pp. 64–76 (1966).

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Further results on evaporating bicomponent fuel sprays

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1. INTRODUCTION

In a recent study [1], the vaporization behavior of a dilute multicomponent fuel spray in a hot laminar airflow was examined. Several liquid- and gas-phase models, which account for the diffusive–convective processes inside and outside the droplet, were investigated. It was indicated that while the effect of transient processes in the liquid is quantitative rather than qualitative for the single-component case, it can cause a more fundamental change in the gasification behavior of multicomponent fuel sprays. This is mainly due to the slow and often rate-controlling liquid mass diffusion process, and due to the volatility differential of the component fuels. It was further demonstrated that the effect of internal liquid motion is less important for the multicomponent case as compared to the single component, since the liquid motion can enhance the mass transport along the streamlines and not across them. The effects of several parameters were studied in the cited paper. However, the influence of three key parameters; namely the liquid Lewis number, volatility differential, and hot air stream temperature was not reported. In this paper, several additional results are presented, which focus on the influence of these parameters. In particular, the sensitivity of the spray vaporization behavior to the three liquid-phase models, namely the diffusion-limit, infinite-diffusion, and vortex models, is further examined as the above-mentioned parameters are varied.

The present study is important because the recommendation of the diffusion-limit model is based on two conditions. One is that the rate of liquid mass diffusion is extremely slow as compared to the droplet surface regression rate. The second is that there exists a substantial volatility differential in order for the liquid mass diffusion to be important. Thus, it is of interest to identify the range of dominant parameters such as the liquid Lewis number, environment temperature, and volatility differential, where the mass diffusion is indeed rate controlling.

2. THE DISCUSSION OF RESULTS

The physical model and the governing equations are described in an earlier paper [1]. Essentially, a transient one-dimensional bicomponent fuel spray in a hot airflow is considered. There are three subsets of equations; namely the gas-phase equations for the gas temperature, species mass fractions, velocity, and density; the liquid-phase equations for the position, velocity, and size of each group of droplets; and the droplet equations which govern the unsteady temperature and liquid mass fractions inside the droplet. These equations are solved by a hybrid Eulerian–Lagrangian explicit–implicit scheme.

NOMENCLATURE

Le	liquid Lewis number
r	non-dimensional radial location in droplet interior
R	non-dimensional droplet radius
t	time
T	initial air temperature
X	axial location in the tube
Y_F	mass fraction of vapor fuel
Y_L	mass fraction of liquid hexane in droplet interior
Y_{LS}	mass fraction of liquid hexane at droplet surface.

For the results presented here, the initial conditions and other properties are essentially the same as in the previous study, except for the variation of the parameter under examination. The temperature of air entering the tube is 1000 K, the tube length is 10 cm, the droplet size of the monodisperse spray is 100 μm, and the slip Reynolds number is about 100. Since the focus of this study is to compare the liquid-phase models, the same gas-phase model, based on the Ranz–Marshall correlation, has been used for all the results. Another model, i.e. the axisymmetric model [1] could also be used. However, the basic conclusions would have remained unchanged.

2.1. *Effect of liquid Lewis number*

The effect of liquid Lewis number (Le) on the predictions of diffusion-limit, infinite-diffusion, and vortex models is portrayed in Figs. 1 and 2. Figure 1 shows the spatial variation of the mass fractions of volatile (hexane) and less-volatile (decane) fuel vapors in the tube for $Le = 1$, 10, 30. Note that in real situations, it would be difficult to change the Lewis number without changing the volatility differential. The purpose here is to isolate the effect of the Lewis number. The results of infinite-diffusion models are obviously independent of the Lewis number, since the internal transport is not considered in the model. An interesting behavior is that the differences between the infinite-diffusion model and the other two models are relatively small at $Le = 1$, but become increasingly significant as Le increases. The implication is that the transient transport processes in the liquid phase have a more significant impact on the spray vaporization behavior for the multicomponent case, because of the large Lewis number, as compared to the single component.

There is another interesting behavior seen in Fig. 1. For both the diffusion-limit and vortex models, at $Le = 1$, the vapor mass fraction of the volatile component is much higher as compared to that of the nonvolatile. Note that the word 'nonvolatile' is used in a relative sense and refers to the less-volatile component. This clearly indicates a batch-distillation type of vaporization behavior. However, as Le increases, the vapor mass fraction of the volatile component tends to become nearly the same as that of the nonvolatile. This means that the vaporization rates of component fuels become nearly the same, thereby indicating that the liquid mass fractions inside the droplet also become nearly uniform, except for a very thin layer near the surface, in the limit of large Le. This is further illustrated in Fig. 2, where the radial variation of liquid mass fraction of volatile component is plotted inside the droplet. Note that, for the purpose of comparison, the axisymmetric distribution (being a function of radius and azimuthal angle) of the vortex model has been transformed into a spherically-symmetric distribution by averaging over spherical surfaces. As seen in Fig. 2, for both models, the mass fraction profiles become nearly constant in the limit of large Le; for the vortex model, the mass fraction profile is not constant near the center, but the region, where it is, constitutes most of the droplet mass. This means that the d^2-law approximation as proposed by Law [2] may be a relevant model for the multicomponent fuel sprays in the limit of large Le.

The major conclusion from the foregoing results is the following. In the limit of $Le = 1$, the spray vaporization behavior appears to be more like batch distillation, as illus-

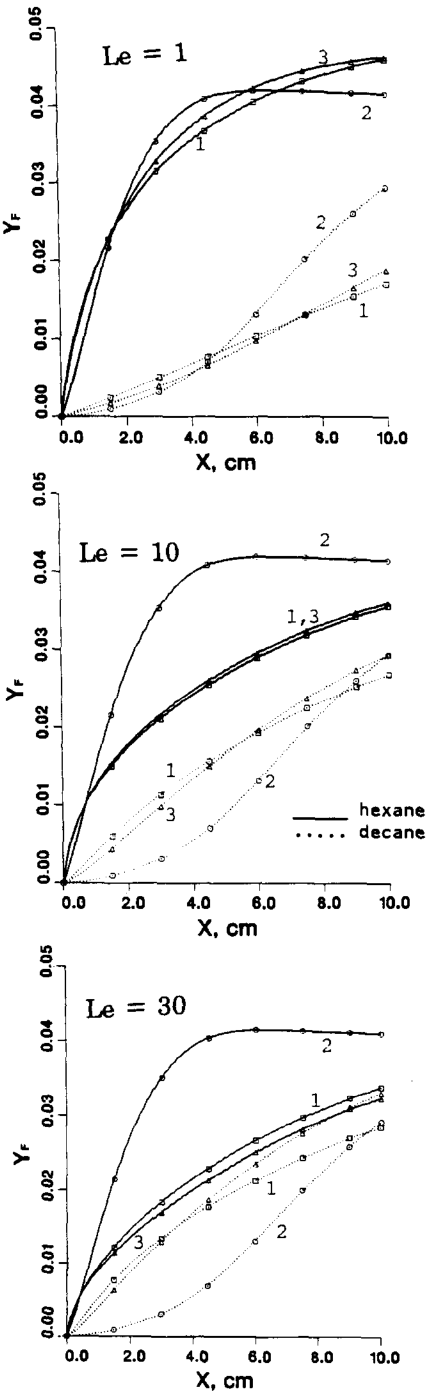

FIG. 1. Profiles of hexane (volatile) and decane (nonvolatile) vapor mass fractions in the tube as predicted by the three liquid-phase models at liquid Lewis numbers of 1, 10 and 30: 1, diffusion limit; 2, infinite diffusion; 3, vortex.

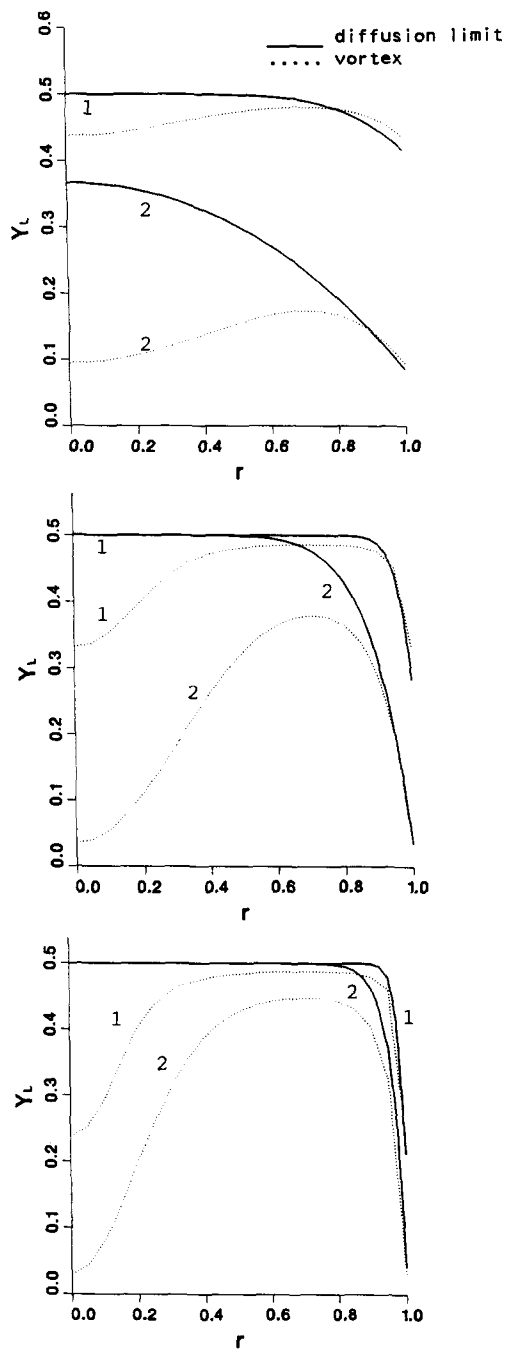

Fig. 2. Comparison of liquid hexane mass fraction profiles at two different times for the diffusion limit and vortex models: 1, 10% mass vaporized; 2, 70% mass vaporized. *Le* = 1, 10 and 30.

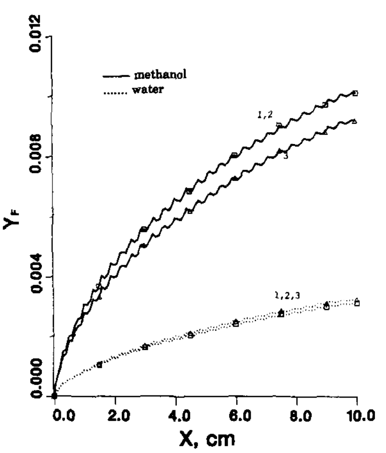

Fig. 3. Vapor mass fractions of methanol and water in the tube as predicted by the three liquid-phase models: 1, diffusion limit; 2, infinite diffusion; 3, vortex. *Le* = 10.

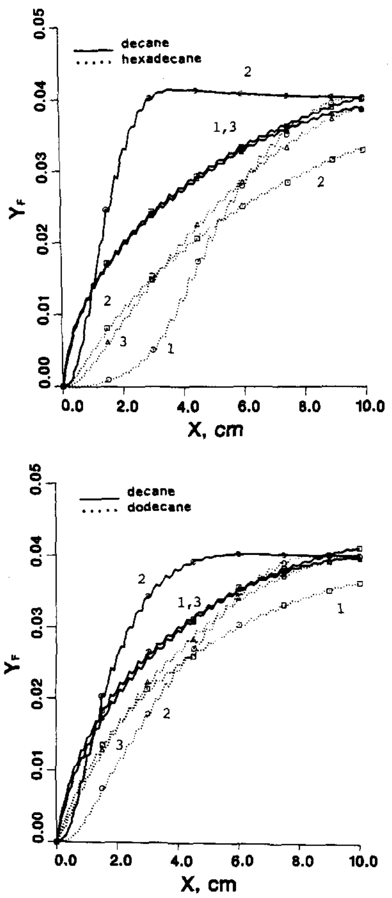

Fig. 4. Vapor mass fractions of volatile and non-volatile components as predicted by three liquid-phase models. *Le* = 10.

trated by the fact that the vapor concentration of the volatile component in the tube is much higher than that of the non-volatile, and that the difference between the infinite-diffusion and the other two models is not significant. At the other limit, when *Le* is large, mass fractions within the droplet become nearly constant for the diffusion-limit and vortex models. This implies that the vaporization rates of volatile and non-volatile fuels become nearly the same, as indicated by the close proximity of the vapor concentration of the two components in the tube. Then the d^2-law approximation of Law [2] would be an acceptable model for predicting the vaporization of multicomponent fuel sprays. Perhaps, a more important conclusion here is that the diffusion limit appears to yield a very consistent and acceptable behavior in the entire range of liquid Lewis number, and is thus recommended.

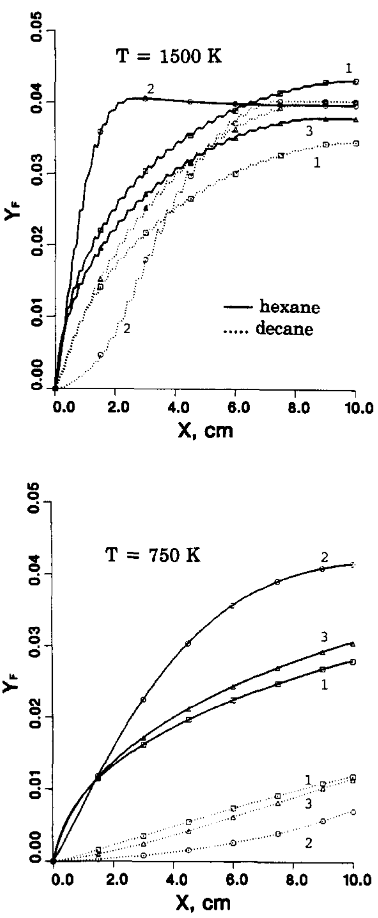

FIG. 5. Vapor mass fractions of hexane and decane in the tube as predicted by three liquid-phase models. $Le = 10$.

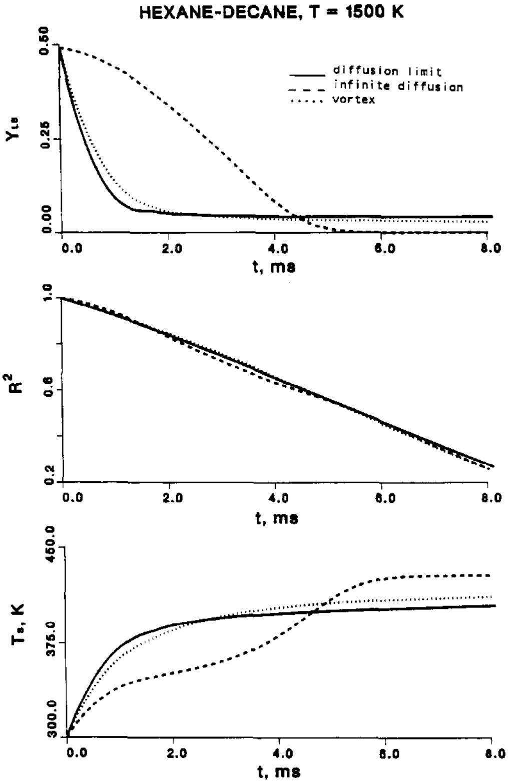

FIG. 6. Surface mass fraction of liquid hexane, surface temperature, and square of non-dimensional radius of a vaporizing bicomponent fuel droplet as predicted by three models: $T = 1500$ K, $Le = 10$.

2.2. *Effect of volatility differential*

In order to examine the effect of volatility differential, a methanol–water mixture is considered. The difference in the boiling temperatures is 35 K as compared to 107 K for the previous case. The Lewis number is assumed to be 10. The heat of vaporization for this mixture is much higher as compared to the hexane–decane mixture. Consequently, the temperature of air entering the tube had to be increased from 1000 to 1500 K in order to have sufficient vaporization. Figure 3 compares the predictions of three liquid-phase models. The important observation is that the spray vaporization behavior is now less sensitive to the models. Note that the predictions of diffusion-limit and infinite-diffusion models are too close to be distinguished in the figure. The observation is further substantiated in Fig. 4, where the results are given for decane–hexadecane and decane–dodecane mixtures. The difference in boiling temperature is 113 K for the first case and 42 K for the second case. As the volatility differential is reduced, the sensitivity to the liquid-phase models, especially the relative difference between the infinite-diffusion model and the other two models, is clearly reduced.

Another point is worth mentioning here. With large volatility differential, there also exists the potential for microexplosion [3]. As discussed in the cited reference, the ideal conditions for microexplosion are that the constituent fuels exist in about equal proportion, and that the liquid Lewis number and volatility differential be sufficiently high. Since such conditions have been considered in the present study, the possibility of microexplosion cannot be discarded. However, the issue has not been addressed and is a limitation of this study.

2.3. *Effect of air temperature*

The preceding results as well as those in ref. [1] indicate a strong sensitivity of the spray vaporization characteristics to the liquid-phase models. In particular, remarkable differences are noticed between the infinite-diffusion model and the other two models. The explanation is that the liquid mass diffusion is the rate-controlling process as it is much slower than the rate of surface regression. It will be interesting, therefore, to examine the behavior by reducing the surface regression rate in order to make it comparable to the rate of liquid mass diffusion. This is done by reducing the temperature of air entering the tube. The predictions of three models for an air temperature of 1500 and 750 K are shown in Fig. 5. The vapor mass fractions, especially those of less volatile components, are much lower for an air temperature of 750 K, indicating the reduction in surface regression rate. However, the relative differences in the infinite-diffusion model and the other two models do not change appreciably, implying that even at low air temperatures the transient mass diffusion in liquid is important. For the spray case, it is not possible to reduce the air temperature much further without changing other parameters, since there is not much vaporization of the non-volatile component in the tube (the residence time of droplets is about 15 ms). In order to further illustrate this aspect, two results for an isolated hexane–decane droplet are presented in Figs. 6 and 7. Note that the average vaporization rate is reduced by a factor of 5 when the air temperature is lowered from 1500 to 500 K. However, the differences between the predictions of the three models remain significant (though somewhat reduced at lower temperature), indicating a strong effect of mass-diffusional resistance in the range of air temperatures considered. Thus, it is hard to conceive a range of ambient temperatures, where the infinite-diffusion model will be a viable model for evaporating multicomponent fuel sprays.

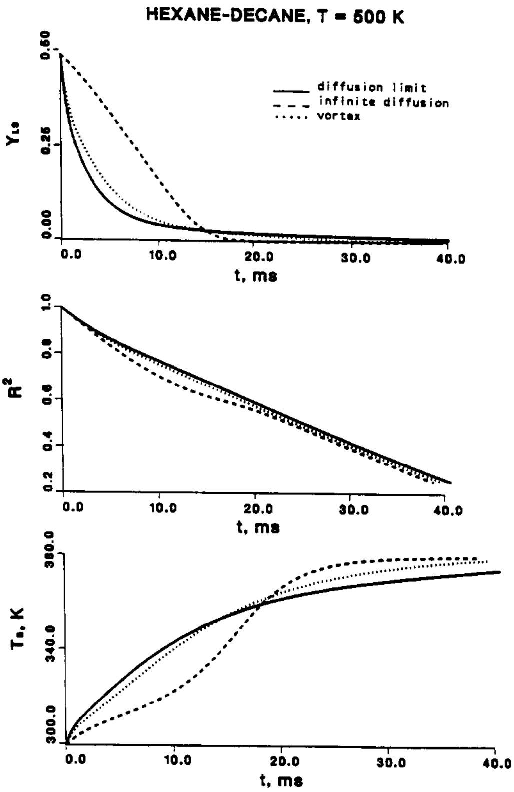

FIG. 7. Surface mass fraction of liquid hexane, surface temperature, and square of non-dimensional radius of a vaporizing bicomponent fuel droplet as predicted by three models: $T = 500$ K, $Le = 10$.

3. CONCLUDING REMARKS

The results given in ref. [1] have been supplemented here in order to illustrate the influence of some key parameters for an evaporating bicomponent fuel spray. The parameters considered are the liquid Lewis number, volatility differential, and the air temperature. The results indicate that the infinite-diffusion model would be a viable approximation where the liquid Lewis number is close to unity or the volatility differential is not large. The d^2-law model [2] for the multicomponent fuel spray may be acceptable in the limit of large Lewis number and volatility differential. The results also indicate that the differences between the diffusion-limit and vortex models are not significant. Based on these results, the diffusion-limit model is again recommended for multicomponent fuel sprays.

REFERENCES

1. S. K. Aggarwal, Modeling of a dilute vaporizing multicomponent fuel spray, *Int. J. Heat Mass Transfer* **30**, 1949–1961 (1987).
2. C. K. Law, Recent advances in droplet vaporization and combustion, *Prog. Energy Combust. Sci.* **8**, 169–199 (1982).
3. C. K. Law, Internal boiling and superheating in vaporizing multicomponent droplets, *A.I.Ch.E. Jl* **24**, 626–632 (1978).